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Astrometric Gravitational Wave Detection

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ABSTRACT
We discuss prospects for the astrometric detection of gravitational waves from massive black hole binaries in the low-frequency regime (∼ nHz). As apposed to beam-like detectors which use delays in light travel times of either lasers (via interferometry; e.g. LIGO) or galactic neutron-star pulses (e.g. the NANOGrav pulsar timing array), astrometric GW detection utilizes the transverse deflections of a photon's trajectory to measure the presence of gravitational perturbations. We construct realistic and comprehensive sensitivity curves for numerous existing and planned astrometric instruments, taking into account not only parallax and proper-motion subtraction but also irregular time-sampling, binarity, and a low-frequency sensitivity boost from down-sampling high-cadence observations. We show that this latter effect will produce a frequency dependence to the astrometric strain sensitivity proportional, $h_{\mathrm{sens}}^{\mathrm{GW}} \propto f^{1/2}$. We expect Gaia to produce the most strongest astrometric GW measurements, at comparable sensitivities to PTA, while WFIRST will be able to probe GW frequencies up to the medium-frequency ($\sim\ 10\mu$Hz), overlapping with the LISA-band. We discuss the exciting complementarity of astrometric detections with those of other types of GW observatories.

Key words: quasars: supermassive black holes

1 INTRODUCTION

The Laser Interferometric Gravitational-wave Observatory (LIGO; Abbott et al. 2009), when it made the first direct detection of gravitational waves (Abbott et al. 2016), ushered us into the era of gravitational wave (GW) astrophysics. LIGO is a beam-detector which uses differences in the light-travel time between two orthogonal detector arms to measure the minute (10^{-23}) perturbations to the local spacetime metric which indicate the presence of a gravitational wave, as predicted by general relativity (Einstein 1916, 1918). LIGO is sensitive to high-frequency GW ($f \sim$ kHz), because of the relatively short light-travel time along its detector arms, which are produced primarily by the merger of stellar-mass black holes and neutron stars. Pulsar Timing Arrays (PTA Estabrook & Wahlquist 1975; Foster & Backer 1990; Jenet et al. 2005) are another beam-type detector which use the naturally precise timings of millisecond pulsars as sensors to constrain, and soon to detect, gravitational waves in the low-frequency regime ($f \sim \mathrm{nHz} \sim \mathrm{yr}^{-1}$) which are believed to be primarily produced by supermassive black holes (SMBH) binaries (Rajagopal & Romani 1994).

In addition to beam-like detectors, which use line-of-sight deviations in timing to measure GW, the alternative strategy of measuring angular deflections in light's path (i.e. transverse to its motion) could also measure deviations to the local gravitational field (Schutz 2010; Book & Flanagan 2011). Gravitational lensing, the same base phenomenon has already been extensively observed, for example the famous multiple-images of distance galaxies produced by massive galactic clusters along the line-of-sight (e.g. Blandford & Narayan 1992). The degree of angular deflection, in the case of gravitational lensing, can be related to the gravitational field of the 'lens',

$$\Delta\theta = \frac{4\,G\,M}{c^2 b} \approx h_{\mathrm{schw}}(r = b) \tag{1}$$

for a lens mass M, gravitational constant G, speed of light c, and impact parameter (i.e. distance of closest approach of the light-ray to the lens) b. If we assume that the same relation applies not only to a Schwarzschild metric h_{schw}, but to a general metric h, for example with a typical low-frequency GW-amplitude $h_{GW}(f \sim 1\mathrm{yr}^{-1} \sim 10^{-15}$, this suggests that the angular displacement of an observed source would be on the order of $\Delta\theta \sim 10^{-10}$ arcsec. Observing such a small angle in a single source (equivalent to the width of a human hair at a distance of the moon) is unlikely

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anytime in the near future. Large astrometric surveys, however, like GAIA (Gaia Collaboration et al. 2016a) could observe a *coherent pattern* of deflections across up to 10^{11} stars in the sky. In this case, the astrometric precision for each source would only need to be, $\Delta\theta \sim h\,(N)^{1/2} \sim 10^{-4}\,''$, which is comparable to the expected precision of GAIA [1].

Recently, Moore et al. (2017) have calculated the first astrometric sensitivity curve, using GAIA-parameters an a method of 'virtual-stars' which they introduce. The authors have shown that the Gaia sensitivity to GW is comparable, to within a factor of ~ 2 to that of current PTA. The authors also demonstrate that unlike beam-like detectors, astrometric GW detectors exhibit sensitivity curves nearly independent of frequency. Beam detectors observe time-delays in light-travel times, which are *integrated* effects over the course of each photon's entire trajectory. In particular, the delay is the integral of GW-induced redshift over a duration $T \sim 1/f$, which means that the strain a detector is sensitive to is linearly proportional to the frequency of the wave. Astrometric displacements, however, are effectively instantaneous, and thus there is no time (or frequency) dependence to their sensitivity (i.e. (1)).

In our analysis, we expand on the sensitivity calculations of Moore et al. (2017) and show that there is, in practice, a $h \propto f^{1/2}$ dependence of astrometric-sensitivity to GW. We also explore the effects of parallax- & proper- motion subtraction, binarity, and irregular time-sampling on realistic astrometric sensitivity curves. We compare the sensitivity of numerous existing and planned instruments to that of PTA, and present an overall scientific case for the benefits of astrometric-GW detection—focusing on useful synergies with PTA for eventually understanding MBH binaries and their evolution.

2 METHODS

To calculate astrometric sensitivity curves, we construct mock observations of stellar positions, inject GW signals with a variety of parameters, and simulate recovering those signals using a realistic detection pipeline. For very low, injected GW amplitudes (far below the eventual detection threshold) the 'recovered' signal is insensitive to the injected signal parameters, and reflects a base 'noise' level. To measure the sensitivity, we increase the injected GW amplitude until the recovered amplitude begins to response: increasing proportionally to the injected signal. To construct a sensitivity curve (vs. frequency), we repeat this procedure for a variety of GW frequencies. We focus on our methodology for the Gaia satellite, and later specify in what ways the procedure differs for different instruments.

To mimic the realistic signal recovery from Gaia (which observes some 10^{11} astrometric positions), we draw some number of stars $N \ll 10^{11}$, and bin them down to a smaller number of 'virtual stars' $M \ll N$, based on a grid of ϕ, θ angular coordinates. The population of stars are drawn randomly from the Gaia first public data release (DR1) catalog (Gaia Collaboration et al. 2016b; Lindegren et al. 2016), which provides coordinates for $\sim 10^{11}$ stars, and the number of times each star was observed over the course of the DR1 observations (~ 418 days between `2014/07/25` [Gaia rev 1078.3795] and `2015/09/16` [rev 2751.3518]). The stellar locations are important for accurately reconstructing the Gaia angular sensitivity, and the number of observations are used to infer typical time-sampling which varies significantly across the sky.

Once initial stellar positions are drawn, they are evolved over time with the addition of parallax- & proper- motions, gaussian noise (independently to each star), and GW-induced angular deviations. For reference, the GW strain from a binary can be written as (Sesana & Vecchio 2010, Eq. 25–27),

$$\begin{aligned} h_{+}^{\mathrm{GW}} &= A^{\mathrm{GW}} \cos\Phi \left(1 + \cos^2\iota\right), \\ h_{\times}^{\mathrm{GW}} &= -\,A^{\mathrm{GW}} \sin\Phi \;(2\cos\iota)\,, \\ A^{\mathrm{GW}} &= 2\frac{(G\mathcal{M})^{5/3}}{c^4 d_L}\,(\pi f)^{2/3}\,, \end{aligned} \tag{2}$$

where the chirp-mass, $\mathcal{M} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$, for a binary with constituent masses m_1 & m_2, phase Φ, orbital inclination ι, and frequency f at a luminosity distance d_L from the observer. We however inject GW with a variety of amplitudes (A^{GW}) and measure the response. The angular deflection can be expressed as (Book & Flanagan 2011, Eq. 58),

$$\delta n^i = \frac{n^i + p^i}{2\,(1 + p_l n^l)} h^{\mathrm{obs}}_{jk} n^j n^k - \frac{1}{2} h^{\mathrm{obs}}_{ij} n^j, \tag{3}$$

where n_i & p_i are unit vectors pointing towards the source of electromagnetic & gravitational radiation respectively, and h^{obs}_{ij} is the spacetime metric at the location of the observer[2]. The spatial components of the GW strain polarization tensors are,

$$h^{\mathrm{GW}} = \begin{bmatrix} h_{+}^{\mathrm{GW}} & h_{\times}^{\mathrm{GW}} & 0 \\ h_{\times}^{\mathrm{GW}} & -h_{+}^{\mathrm{GW}} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

for a wave traveling along the $\hat{z}$ axis.

We use the method of 'virtual stars' described by Moore et al. (2017) to analyze simulated astrometric GW measurements. With this method, the sky is grouped into angular bins and the motion of all stars within a bin are averaged together into displacements of a much smaller number of virtual stars over the sky. We then calculate coefficients for the spin-weighted spherical harmonics of the averaged displacements, and fit sinusoids to those coefficients over time which encode the GW parameters. In practice, we design the angular bins to be easily transformed into spherical harmonics using Mike Boyle's code `spinsfast`[3] based on the algorithm of Huffenberger & Wandelt (2010).

3 INTERMEDIATE RESULTS & CONCLUSIONS *[IN PROGRESS]*

A Gaia sensitivity curve is shown in Fig. 1, including the effects of proper- and parallax- motion (and subsequent subtraction). The low-frequency drop in sensitivity is produced primarily by proper-motion subtraction, while the localized decrease in sensitivity at $f \sim 1\,\mathrm{yr}^{-1}$ is due to parallax. Overall our sensitivity curve is consistent with that of Moore et al. (2017), and notably we also find the sensitivity to be independent of frequency within the Nyquist band, defined as the frequency range $1/T < f < 1/\delta t$, for a total observing duration T, and observing cadence δt (denoted by

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[1] While one might naively expect a $\sim$ 100 m telescope aperture to be required for 10^{-10} angular resolution, spatial-scanning can be used to drastically improve angular localization beyond the standard diffraction limit for a single image (e.g. Riess et al. 2014).

[2] We use Einstein-notation for summation over repeated indices, e.g. $p_l n^l \equiv \vec{p} \cdot \vec{n}$.

[3] https://github.com/moble/spinsfast

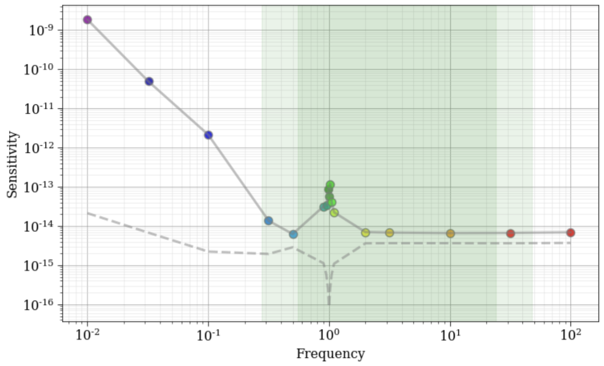

Figure 1. Characteristic-strain sensitivity versus GW frequency (in units of yr^{-1}) for a simulated Gaia detection pipeline. This model includes the inject and subtraction of proper and parallax motion, the latter of which produces the drop in sensitivity at $f \sim 1\,\mathrm{yr}^{-1}$. In this model it is assumed that the observations of all stars are synchronized. No binary motion is injected or subtracted.

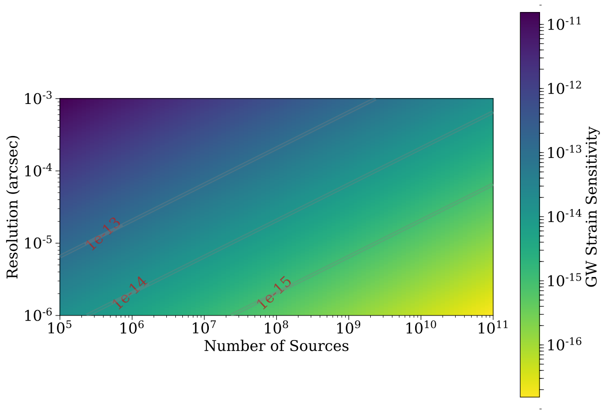

Figure 2. GW strain sensitivity achieved with a given angular resolution (in arcseconds) in the given number of observed sources. Gaia, with an angular resolution of $\sim 10^{-3}\,''$, observing $\sim 10^{11}$ stars, obtains a sensitivity of roughly 10^{-14}. A survey observing with an angular resolution of $10^{-6}\,''$, for example by VLBI, could observe 10^{7} sources (e.g. quasars) and obtain a sensitivity of $\sim 10^{-15}$.

the green shaded region). At frequencies above the upper sampling limit ($f > 1/\delta t$), the sensitivity will again drop, due to interpolation when comparing between different stars and regions of the sky with different time sampling characteristics. This effect isn't yet implemented in our analysis.

Figure 2 shows the landscape of GW strain sensitivities obtained for different survey characteristics, parametrized as angular resolution versus number of sources observed. Gaia, with an angular resolution of $\sim 10^{-3}\,''$, observing $\sim 10^{11}$ stars, obtains a sensitivity of roughly 10^{-14} ((1)). The observed sources do not need to be restricted to stars, however, as quasars could also be used. Observing quasars with VLBI offers the advantage of increased angular resolution, as high as $10^{-6}\,''$. If continued and high-SNR monitoring of AGN sources were to yield angular position accuracies of 10^{-7} arsec, then only 10^{5} sources would be required to obtain a sensitivity of $\sim 10^{-15}$.

One benefit of ground-based GW detectors in the low-frequency regime is the ability to support longer duration missions. For Gaia, with a planned nominal duration of ~ 5 yr, typical stars will be observed ~ 350 times, and the lowest sensitive frequency will be $f \sim 0.2\,\mathrm{yr}^{-1}$. The most targeted source of low-frequency GW is the stochastic GW background (GWB) produced by the incoherent superposition of many MBH binaries, which is expected to scale as (Phinney 2001),

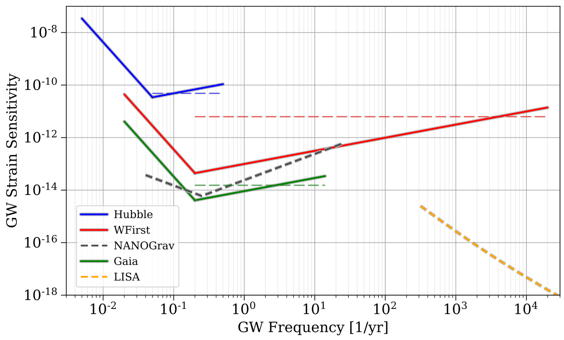

Figure 3. GW strain sensitivity vs. frequency for astrometric GW detection using Hubble (blue), WFIRST (red), and Gaia (green) observations, in addition to a sample PTA sensitivity curve (NANOGrav, black) and LISA curve (orange). The blue, red & green dashed lines show the sensitivity as given by the sensitivity scaling relation (1), while the solid lines include the low-frequency turn-off from proper motion fitting, and the improvement from binning high-cadence observations which produces the $h_{\mathrm{sens}}^{\mathrm{GW}} \propto f^{1/2}$ dependence. Gaia is able to reach a sensitivity remarkably similar to that of current PTA, while WFIRST—with an exceedingly high cadence—probes GW frequencies into the LISA band. Note: the astrometric and LISA sensitivity curves are *ideal* while that of NANOGrav is *empirical*.

$$h_c^{\mathrm{GWB}} \sim A_{1\mathrm{yr}^{-1}}^{\mathrm{GWB}} \left(\frac{f}{1\,\mathrm{yr}^{-1}}\right)^{-2/3}, \tag{5}$$

where typical estimates place the amplitude at $f = 1, \mathrm{yr}^{-1}$ near $A_{1\mathrm{yr}^{-1}}^{\mathrm{GWB}} \sim 10^{-15}$. As the observing duration is extended, the increased number of observations beat down the noise, but also a lower-frequency of the GWB is probed with a higher intrinsic amplitude. In effect, the GWB strain probed by an astrometric-gw observatory should scale roughly as, $h_c^{\mathrm{GWB}}/h_{\mathrm{sens}}^{\mathrm{GW}} \propto T^{7/6}$. Increasing the observing duration to 20 yr, then increases the effective sensitivity by roughly a factor of 5.

As previously mentioned, incorporating additional observations can be used to increase the precision of astrometric positions. For example, when searching for a GW with a period of ~ 1 yr, observations separated by less than something like ~ 0.1 yr could be simply binned together to decrease the noise between observations. In this way, while the intrinsic sensitivity of the astrometric GW detection method is insensitive to frequency, the effective sensitivity can be improved in a frequency dependent manner based on the detection pipeline. Fig. 3 show the sensitivity scaling relation from (1) (dashed lines) combined with the low-frequency dependence of the full calculation (i.e. Fig. 1), and including the effects of binning high-cadence observations.

In the case of WFIRST (e.g. Spergel et al. 2015), where the microlensing survey expects to observe $\sim 10^{7}$ stars in the bulge at a cadence of ~ 15 min, for a duration of ~ 5 yr, the sensitivity boost from down-binning up to 10^{5} images becomes very significant. The high-cadence observations themselves produce the ability to probe GW frequencies as high as $10^{3} - 10^{4}\,\mathrm{yr}^{-1}$. This means that LISA and WFIRST could potentially detect the same, sufficiently loud, binary single-sources. Astrometric detection with Gaia is likely to be about an order of magnitude more sensitive that WFIRST owing to their larger number of observed stars, and better intrinsic astrometric precision. The Gaia sensitivity curve (green) is actually markedly similar to that of current PTA (dashed black line), with slightly better high frequency sensitivity (owing to its weak frequency sensitivity) and poorer low frequency (due to shorter observing duration).

There are numerous possible synergies to observing the same GW sources with numerous detection methods. While LIGO (and now LIGO-VIRGO) requisitely makes detections with multiple detectors, and a slew of on-sight and on-line validation mechanisms to remove false positives, a PTA detection—especially of the GWB—would be more difficult to independently 'confirm'[4], especially as the expected residual from a true GW signature is intrinsically similarly to an excess of red-noise in the pulsars or detection pipeline. While there is no reason to expect PTA to suffer from issues of false positives, an entirely independent confirmation of the same signal would obviously be very helpful for validating the methodologies on both sides. PTA and astrometric detections are also nicely complementary in their 'beam-patterns' i.e. their spatial sensitivity distributions. Because PTA rely on line-of-sight effects, while astrometric detection is orthogonal to the light-ray, their beam patterns are nearly inverted. PTA observe pulsars preferentially in the galactic plane and towards the galactic center (where pulsars and especially millisecond pulsars tend to be found) meaning their sensitivity tends to points in the same direction. Gaia also tends to observe stars in the galactic plane and center, but this produces peak sensitivities out of the plane.

Gaia is planned to have a lifetime of ~ 5 yr, while PTA have already collected data for a baseline of ~ 20 yr and are expected to continue indefinitely. PTA will thus necessarily be more sensitive than Gaia (likely immediately, but at least in the future). Because of the drastic difference in the numbers of 'detectors' ($\sim 10^2$ pulsars for PTA; and $\sim 10^{11}$ stars for Gaia), however, astrometric detectors will have an angular resolution—theoretically able to reach nearly 1 – 10 arcsecond resolution on the sky—drastically better than both PTA and even LISA. Even in the case of a PTA GWB detection and Gaia non-detection, the Gaia measurements may still be able to provide valuable constraints on the anisotropy of the signal.

In addition to the stochastic GWB, individual, sufficiently loud, massive black hole binaries will also be observable as foreground sources (e.g. **?**) which are expected to be more dominant at higher frequencies. While it is likely that PTA will be more sensitive to the GWB than Gaia, the astrometric method may win out in detecting single sources at higher frequencies. Gaia's significantly increased angular resolution of GW signals would then be additionally useful for identifying candidate MBH binary host-galaxies, and possibly finding an electromagnetic counterpart to the GW signal.

4 SUMMARY

This paper summarized the current status of our study on astrometric gravitational wave (GW) detection. Following the methods of Book & Flanagan (2011) & Moore et al. (2017) we have shown that Gaia may be able to reach GW sensitivities competitive with pulsar timing arrays (PTA), the current state of the art in the low-frequency GW band. We have shown that proper- and parallax- motion subtraction has only weak effects on astrometric sensitivity, and we are in the process of demonstrating the same for binarity and the irregular time-sampling used by Gaia. Astrometric GW observations thus seems like a highly viable detection pathway to complement LIGO/LISA/PTA beam-like detectors.

While the intrinsic astrometric response to GW signals is achromatic, we show that plausible detection pipelines will be able to boost their low-frequency sensitivity producing a $h^{\mathrm{GW}}_{\mathrm{sens}} \propto f^{1/2}$ dependence. While Gaia will likely have a much higher overall sensitivity, the high-cadence observations of the WFIRST microlensing survey will be able to probe GW signals in the medium-frequency regime ($10 - 100\,\mu$Hz) overlapping with LISA. Having astrometric detecters coordinated with PTA at low frequencies, and LISA at high frequencies, offers exciting opportunities to leverage each's scientific abilities. At the most basic level, coincident detections will allow the validation and detailed calibration of each method. Because astrometric detecters have different frequency sensitivities, it may be possible to observe individual sources moving across the band from PTA to Gaia, or from WFIRST to LISA, offering constraints on the evolution of not only populations but individual binary systems. While PTA and LISA will each be more sensitive overall than astrometric detectors, the angular localization capabilities by Gaia (for example) will be unparalleled in measuring anisotropies and sky locations of GW signals.

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This research made use of `Astropy`, a community-developed core Python package for Astronomy (Astropy Collaboration et al. 2013), in addition to `SciPy` (Jones et al. 01), `ipython` (Prez & Granger 2007), `NumPy` (Van Der Walt et al. 2011). All figures were generated using `matplotlib` (Hunter 2007).

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[4] Note, for example, that the different PTA have significant overlap in the pulsars they observe.

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