Novel tests of general relativity

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We present a useful parameterization and a flexible model for the effects of calibration errors in gravitational wave detections on measured gravitational waveforms.

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I. INTRODUCTION

The general relativity (GR) has passed a multitude of tests over the past years [M. Will et al. 2014], but it has never been matched up against a strong gravitational field, like that of a black hole. The detection of gravitational waves (GWs) emitted from binary black hole merger [B. P. Abbott et al. 2016 & 2017] gives us access to a genuinely strong gravitational field regime. In order to test the GR model beyond the currently accessible scale, we will focus on quantifying the deviations between the observed waveform (h_{obs}) and the waveforms modeled by GR (h_{GR}). Quantifying such deviations is one of the major challenges in GW data analysis.

The aim of the project is to find a way of characterizing/parameterizing deviations from GR waveforms that makes little prior assumptions about the nature of the deviation. The approach can thus be viewed as data orientated rather than testing a particular strawman model. The numerical method we use to characterizing/parameterizing deviations is based on spline interpolation in which the estimated signal is a cubic spline function.

The splines employed to characterize the deviation provide a uniform way of describing GR departures rather than fitting distinct models to the inspiral and ringdown parts of the waveform. Provided the template basis describing the GR waveform is accurate enough this should provide a good way of describing departures at the transition from inspiral to merger.

II. DESCRIPTION OF THE METHOD USED TO CALIBRATION ERRORS

A. Waveform Representation

When a gravitational wave with a frequency-domain waveform $h_{\rm GR}(f)$ enters to the detector, we assume it records a data stream (again in the frequency domain) that is an additive combination of a waveform and noise:

$$d(f) = h_{\rm obs}(f) + n(f). \tag{1}$$

Because the detector is not perfectly calibrated, however, there are frequency-dependent amplitude and phase departures in $h_{\rm obs}$ with respect to $h_{\rm GR}$:

$$h_{\rm obs}(f) = h_{\rm GR}(f)\left[1 + \delta A(f)\right] exp\left[i\delta\phi(f)\right]. \tag{2}$$

Despite those departures are small, have the potential to impact in the measurement of all parameters of the source (masses, spins, distance, sky location, etc.). Therefore, a careful analysis is mandatory to evaluate the effect of deviations. Under the assumption of $\delta A(f)$ and $\delta\phi(f)$ oscillates smoothly in frequency, they can be modeled by a spline function.

B. Spline Model

A spline function is a piecewise polynomial interpolation that obeys smoothness conditions at the nodal points where the pieces connect. In the following, we use the case of cubic splines defined by a 15 fixed nodal points confined to a finite frequency interval. Formally these departures can be written as

$$\delta A(f) = p_3(f; \{f_i, \delta A_i\}), \tag{3}$$
$$\delta\phi(f) = p_3(f; \{f_i, \delta\phi_i\}), \tag{4}$$

where p_3 is a cubic spline polynomial, the f_i are the nodes of the polynomial in frequency, and δA_i and $\delta\phi_i$ are the values of the spline at those nodal points. The parameters of this model are then the δA_i and the $\delta\phi_i$. Each detector will have independent calibration parameters in a multi-detector analysis. As [S. Vitale et al. 2012] did it, we will choose nodal points equally spaced in the $\log f$, this choice constrains the correlation length of the calibration errors in frequency space.

Because the calibration errors are expected to be small, it seems reasonable to place a Gaussian prior on the calibration error parameters

$$p(\delta A_i) = N(0, \sigma_A), \tag{5}$$
$$p(\delta\phi_i) = N(0, \sigma_\phi), \tag{6}$$

where σ_A and σ_ϕ characterize our expected uncertainty about the magnitude of the calibration error at these frequencies. These parameters can then be fit and the corresponding calibration errors marginalized over in a run of one of the LALInference samplers.

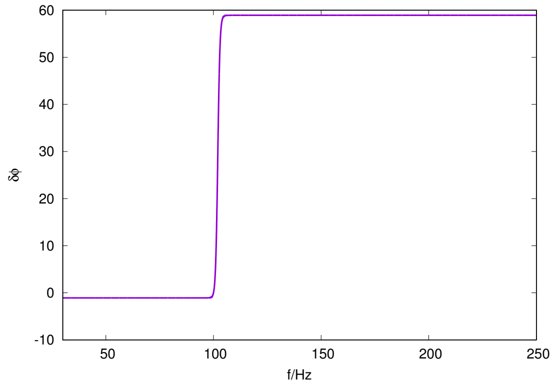

FIG. 1: Injected modified $\delta\phi$.

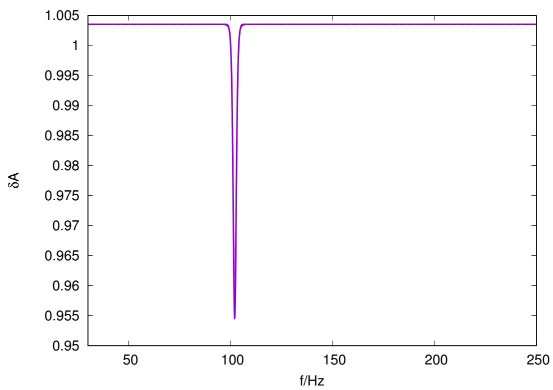

FIG. 2: Injected modified δA.

We would expect departures from GR to occur in regions other than the limits already imposed by Binary pulsar observations. Alternately viewed we expect GR to hold in a manner similar to the way it does in binary pulsar systems in the early inspiral parts of the waveform, placing strict constraints on the departure of the amplitude spline in this region, basically restricting it to having an average value close to zero, thus removing the amplitude degeneracy.

III. PRELIMINARY RESULTS

In order to test the spline, we take the range of frequencies to be $[30, 250]$ for an injected artificial modified GR signal, the reason why this election was made is because we want to reflect the GW150914 bandwidth. The modified GR signal was not based on a physical model, but invented to mimic spontaneous scalarization. For the phase $\delta\phi$, we added an abrupt change centered at $f = 102$Hz, with a width of ~ 1Hz. This change is of 60 radians. The amplitude temporarily dropped by 5%. In Figs 1 and 2 we show the injected modified phase and amplitude, respectively.

These modifications in the amplitude and the phase generate a modified signal, which we added Gaussian

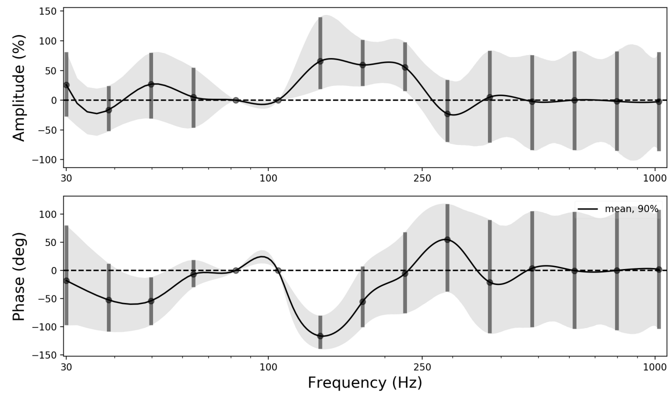

FIG. 3: The posterior probability of amplitude and phase of the modified waveform. In a confidence interval of 90%.

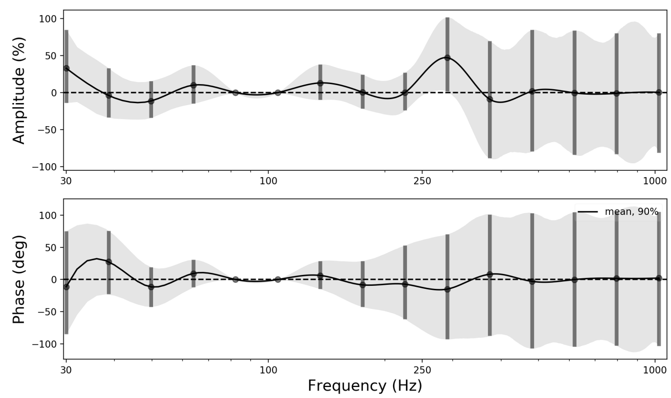

FIG. 4: The posterior probability of amplitude and phase of the waveform predicted by GR. In a confidence interval of 90%.

noise, this new signal with noise was analyzed by our model and later was compared with a signal predicted by the GR model. Fig. 3 shows the amplitude and phase of the modified signal evaluated at the nodal in a confidence interval of 90%, in this cases the uncertainties in amplitude and phase are $\sigma_A = 5\%$ and $\sigma_\phi = 60$ degrades, respectively. In order to remove the degeneracy in amplitude we fixed two nodal points, without loss of generality, we fixed these two nodal points around 100Hz. In this figure, we can see, clearly, deviation of the GR model in the amplitude and phase of the modified signal.

In a similar way, Fig. 4 shows the amplitude and phase of the signal predicted by GR evaluated at the nodal in a confidence interval of 90%, the uncertainties in amplitude and phase were the same of the modified signal. Also, we fixed the same two nodal point like in the modified signal. In this case, the phase does not show departures of GR model, as it was expected. However, in the amplitude there is a point, after 250Hz, that departures from the GR model, this is undoubtedly a reason for detailed analysis to determine the cause of such departure.

In order to see how these deviations from the GR model

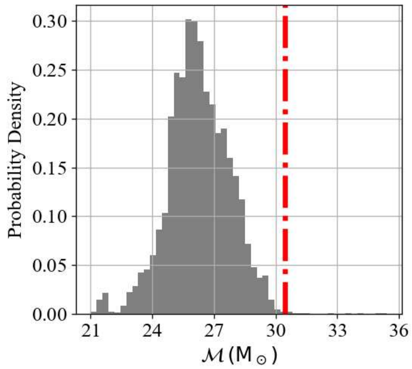

FIG. 5: The posterior probability of chirp mass with the modification to GR present.

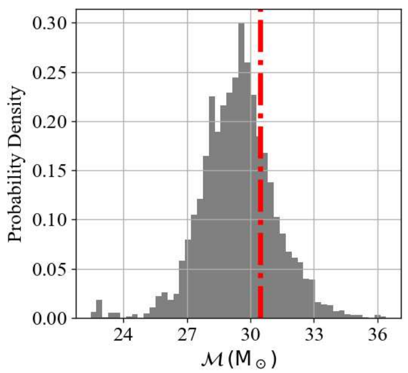

FIG. 6: The posterior probability of chirp mass without the modification to GR present.

affect the measurements of the observable quantities, we show the recovered chirp mass distribution by the two signals mentioned above. In Fig. 5, we show the posterior probability of chirp mass of the modified signal, where there is a departure of ~ 4 solar masses.

Fig. 5 shows the posterior probability of chirp mass of the signal modeled by GR. The expected value is within the probability density.

IV. FINAL COMMENTS

We have presented a numerical algorithm based on splines to find a way of characterizing/parameterizing deviations from GR waveforms. In order to carry this out, we injected two signals with a Gaussian noise present them. On the one hand, the amplitude and phase of one of these signals was modified by a spontaneous scalarization, this signal showed deviations from the GR model. On the other hand, one of the injected signals was a predicted signal by GR. The phase of this signal shows zero deviation from GR, while the amplitude did it. A careful study is needed to determine the causes of this deviation. As well as: (i) determine in what way these deviations could be confused with calibration errors, (ii) implement another modified waveform models, and (iii) improve the way to remove the amplitude degeneracy.

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[M. Will et al. 2014] C. M. Will, Living Rev. Rel. 17, 4 (2014).

[B. P. Abbott et al. 2016 & 2017] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. 116, 061102 (2016). B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. X6, 041015 (2016). B. P. Abbott et al. (VIRGO, LIGO Scientific), Phys. Rev. Lett. 118, 221101 (2017). B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. 116, 221101 (2016).

[N. Yunes et al. 2016] N. Yunes, K. Yagi, and F. Pretorius, Phys. Rev. D94, 084002 (2016).

[S. Vitale et al. 2012] S. Vitale, W. Del Pozzo, T. G. F. Li, C. Van Den Broeck, I. Mandel, B. Aylott, and J. Veitch, Phys. Rev. D 85, 064034 (2012).