Notes about wind in binaries

September 29, 2017

1 Analytic stuff

1.1 Orbit evolution

In an eccentric binary with two stars of mass m_1 and m_2, reduced mass μ, total mass m_t and semimajor axis a, the total angular momentum is given by

$$J = \mu r V_{orb}(r) \tag{1}$$

V_{orb} is the total velocity of the two stars, and it is a function of the separation r between them, as they move through the orbit

$$V_{orb}^2 = Gm_t \left(\frac{2}{r} - \frac{1}{a} \right) \tag{2}$$

The angular momentum is conserved throughout the orbit. To get the value of J just pick a value for r, like the periaps radius $r_p = a(1-e)$, then you get J^2 equal to

$$J^2 = \frac{(m_1 m_2)^2}{m_t} Ga(1-e^2) \tag{3}$$

To understand how mass transfer can change the angular momentum, we want the derivative of J. But it is a lot simpler to take the derivative of J^2, so we rewrite $\dot{J}$ in therms of $(\dot{J^2})$:

$$(\dot{J^2}) = 2J\dot{J} \rightarrow 2\frac{\dot{J}}{J} = \frac{(\dot{J^2})}{J^2} \tag{4}$$

Using this relation we can get analyze how the orbit evolves from the total derivative of J^2.

J^2 consists of four factors. The derivative of each factor is :

$$\begin{aligned} \left((m_1 m_2)^2\right)' &= 2m_1\dot{m}_1 m_2^2 + 2m_2\dot{m}_2 m_1^2 = 2(m_1 m_2)^2 \left(\frac{\dot{m}_1}{m1} + \frac{\dot{m}_2}{m2} \right) \\ \left(\frac{1}{m_t} \right)' &= -\frac{\dot{m}_t}{m_t^2} \\ a' &= \dot{a} \\ (1-e^2)' &= -2e\dot{e} \end{aligned} \tag{5}$$

Put these together using the chain rule, and get the full derivative:

$$2\frac{\dot{J}}{J} = \frac{(\dot{J^2})}{J^2} = 2\left(\frac{\dot{m}_1}{m1} + \frac{\dot{m}_2}{m2}\right) - \frac{\dot{m}_t}{m_t} + \frac{\dot{a}}{a} - \frac{2e\dot{e}}{1-e^2} \tag{6}$$

This is the equation we can use to understand how changes in mass and angular momentum will affect the orbit.

1.2 Orbit evolution for binaries with winds and torques

We want to understand how the orbit evolves for a binary, where one donor star is loosing mass as a wind from its surface. The companion accretes some of the gas and shapes motion the motion of the rest. To use equation 6 on this specific system, we first set some definitions.

To simplify the problem we assume that the orbit is circular. This removes the influence of eccentricity.
Then to keep track of the mass loosing star and the accreting companion, we change m_1 to m_d for donor star, and m_2 to m_a for accreting star. The donor star is loosing mass $\dot{m}_d$. The companion accretes some fractions β of the mass lost from the donor, so $\dot{m}_a = -\beta\dot{m}_d$. β is determined from the simulations.

We also need to keep track of the change in angular momentum. The angular momentum lost in the wind from the donor star is $\dot{J}_d = \dot{m}_d(r_d \times v_d)$, which is the angular momentum the gas had, while it still was part of the star. In a circular orbit the velocity of the star is always perpendicular to its radial vector, so we can drop the crossproduct and write $\dot{J}_d = \dot{m}_d\ r_d\ v_d$. Remember that $\dot{m}_d$ is negative, so $\dot{J}_d$ is negative.
Some of the angular momentum lost from the donor star will be returned to the orbit, when the companion accretes the gas. This adds a factor $\dot{J}_a$ to the total change. The angular momentum of the accreted gas will have changed as it moved across the roche potential of the two stars, so this factor will have to be measured numerically.
The gas that is not accreted by the companion, is left around the binary. Some of it is focused in wakes behind the two stars, and these wakes exert dragforces on each object. This will transfer an angular momentum $\dot{J}_{drag}$ from the orbit to the wakes. How great this effect is, depends on how much gas is present around the binary, and how fast the wind is moving. So this factor also has to be measured numerically, but is definitely negative.
In total the change in angular momentum is $\dot{J} = \dot{J}_d + \dot{J}_a + \dot{J}_{drag}$.

To make it easier to understand how $\dot{J}$ will affect the orbit, we introduce the factor α, defined as:

$$\alpha = \frac{\dot{J}_d + \dot{J}_a + \dot{J}_{drag}}{\dot{m}_d r_d v_d} = \frac{\dot{J}_d + \dot{J}_a + \dot{J}_{drag}}{\dot{J}_d} \tag{7}$$

so that α represents the extra change in angular momentum measured from the simulations. This gives the total angular momentum $\dot{J} = \alpha\ \dot{m}_d\ r_d\ v_d$. Putting all this into equation 6, we get the orbit evolution:

$$2\frac{\alpha\ \dot{m}_d r_d v_d}{\mu r V_{orb}} = 2\left(\frac{\dot{m}_d}{m_d} + \frac{-\beta\dot{m}_d}{m_a}\right) - \frac{\dot{m}_d - \beta\dot{m}_d}{m_t} + \frac{\dot{a}}{a}$$

To simplify a bit we can substitute $r_d = \frac{m_a}{m_t}a$ and $v_d = \frac{m_a}{m_t}V_{orb}$, because of the orbit is circular, and we set the mass ratio $q = \frac{m_d}{m_a}$, so $m_t = m_d(1+\frac{1}{q})$. Factoring out $\frac{\dot{m}_d}{m_t}$ the change in separation follows

$$\begin{aligned}\frac{\dot{a}}{a} &= 2\frac{\alpha\ \dot{m}_d \frac{m_a}{m_t} a \frac{m_a}{m_t} V_{orb}}{\frac{m_a m_d}{m_t} a V_{orb}} - 2\left(\frac{\dot{m}_d}{m_d} + \frac{-\beta\dot{m}_d}{m_a}\right) + \frac{\dot{m}_d - \beta\dot{m}_d}{m_t} \\ &= 2\frac{\alpha\ \dot{m}_d}{q m_t} - \frac{\dot{m}_d}{m_d}(2 - 2q\beta) + \frac{\dot{m}_d}{m_t}(1-\beta) \\ \rightarrow \frac{\dot{a}}{a} &= \frac{\dot{m}_d}{m_t}\left(2\frac{\alpha}{q} - 2\left(1+\frac{1}{q}\right)(1-\beta q) + (1-\beta)\right) \end{aligned} \tag{8}$$

$\dot{m}_d$ is negative, so if the sum in parentheses turns out negative as well, $\dot{a}$ must be positive and the orbit will widen. If the paranthesis turns out positive, $\dot{a}$ will be negative and the orbit will shrink.

2 simulation stuff

For the simulations we use the eularian code Athena++ Athena++ has the nice feature of allowing the user to work in a spherical grid, solving the equations of hydrodynamic in a co-rotating frame. The grid conserves dr/r for cell size in the radial direction. This means that the inner most cells of the sphere are smallest, and the farther away from the center the bigger are the cells.
We place our donor star at the center, where the refinement is highest. This creates the inner boundary at $r_{\min} = r_{\text{donor}}$. The companion is placed at a seperation $a = 1$, so the length unit becomes a. The mass scale is set by the total mass $m_d + m_a = m = 1$. We use a constant $\dot{m}_d$ for all simulations.

The cell size at the companion has to be refined enough to resolve a gas focusing length of the star

$$r_f \sim \frac{2Gm_a}{v_{wind}^2} \tag{9}$$

To avhieve this we add to extra levels of refinement at the position of the companion. The grid structure seen for simulations of wind velocities with inner boundary slightly larger than the roche lobe in can be seen in figure 1.

3 Results

For a simplified case with an equal mass binary ($q = 1$) and no accretion on the companion ($\beta = 0$), eq. 8 simplifies to

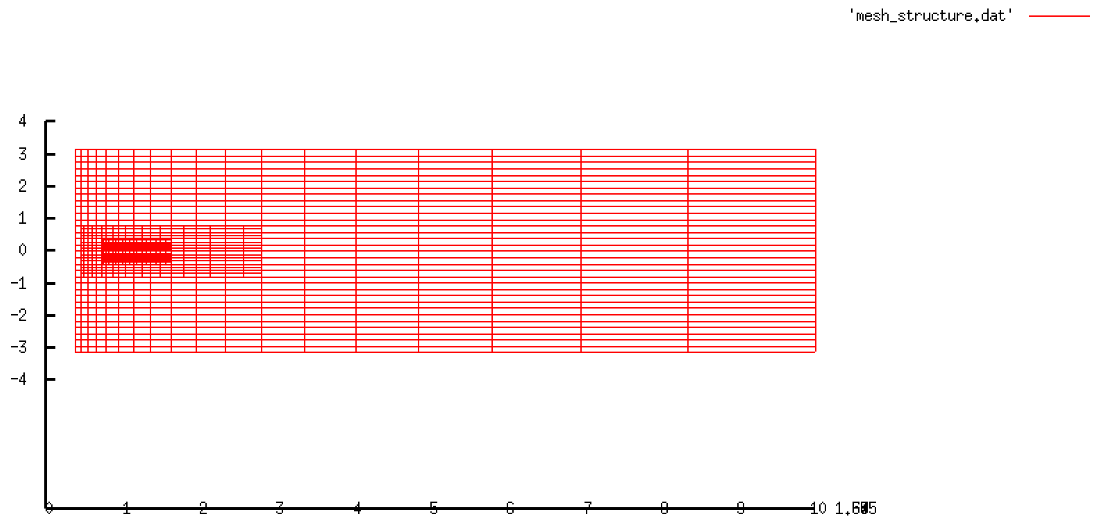

Figure 1: Mesh structure for 2D smr runs with $r_{min} = 0.38$. x-axis is radial coordinate, y-axis is phi coordinate. Imagine it wrapped around, so the left side is the innermost cells around the donor star, and the right side is the outer edge. The patch with extra refinement is at the location of the companion.

$$\frac{\dot{a}}{a} = \frac{2\dot{m}_d}{m_t}\left(\alpha - \frac{3}{2}\right) \tag{10}$$

where the contributions to α now only includes angular momentum lost from the donorstar and torques from gas. The analytical solution found from assuming no torque from gas material ($\alpha = 1$) is called the $Jeans' mode$. It shows that binaries losing mass through winds will always grow apart.

The flow structure very much depends on the wind velocity compared to the orbital velocity. For a fast wind (or a very large separation), the timescale for gas to leave the binary is much faster than the orbital timescale. The gas will only interact with a companion in a manner similar to bondi-hoyle accretion with a wake behind the companion.
For a slow wind the gas is present in the binary on timescales comparable to the orbit, so wind velocity $\sim$ orbital velocity. The material is then much more affected by both stars as described in section 1.1.
As we decrease the wind velocity to be comparable to orbital velocity, we pass below the escape velocity of the binary. The gas material is trapped and more material is present around the binary to create dragforces. The different flowstructures are seen in figure 2.

For the wind simulations the gas enters the grid from the center and leave the grid, when it passes beyond $r = 10a$. In figure **??** is plotted the total mass present in the grid with time. In the first period, 2π, gas is injected and non is moving fast enough to leave, so

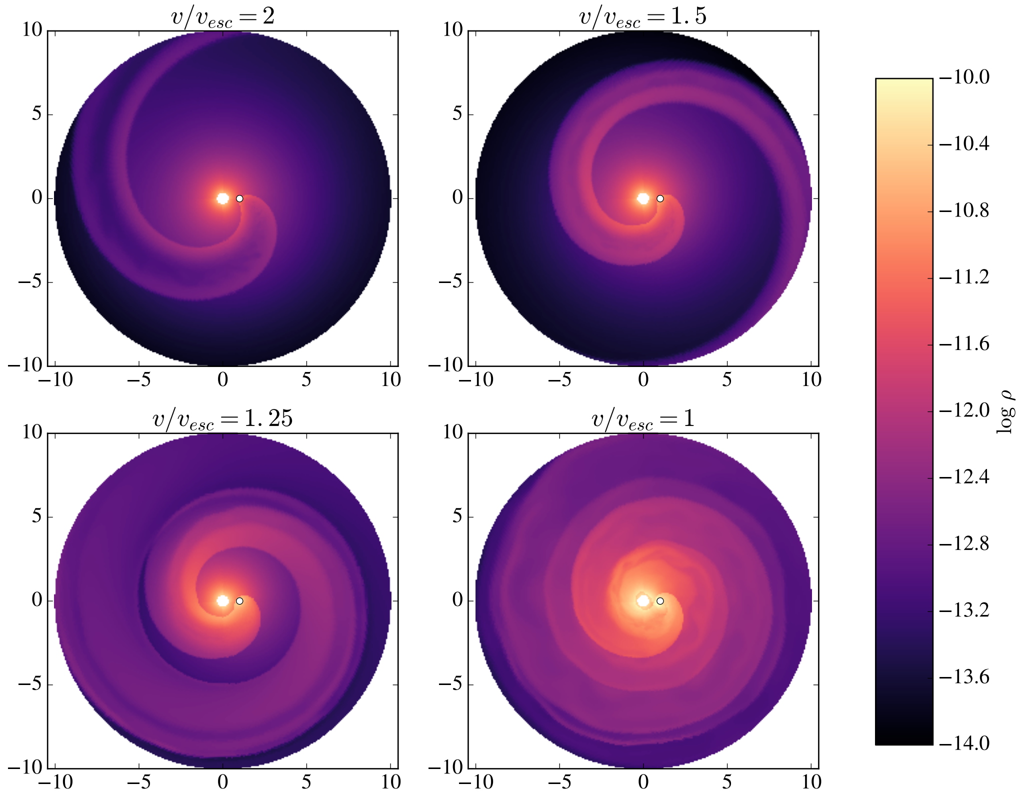

Figure 2: Density plot for different wind velocities. As we decrease the wind velocity the spiral shaped wake wends more tightly around the binary. For $v/v_{esc} = 1.25$ the velocity is below escape velocity of the binary, so the wake "bites its own tale". The momentum of the emitted gas is not high enough to push away material from the system.

they follow the black line indicating all mass injected. For winds above escape velocity the amount of mass entering and leaving reaches steady states, either as the wind escapes the binary (fast wind) or as enough material is build of around the binary to have pressure pushing gas away (slow winds). For the case of $v/v_{esc} = 0.5$, the gas doesn't have enough energy to even escape the donor star. The material falls back through the inner boundary, and is lost from our calculations. The simulations for wind below escape velocity if the star is therefore not included further (even though they look really pretty :-)).

Figure 4 shows the total torque on the stars from the gas for different wind velocities. Also plotted is the total angular momentum lost from the donor. This is constant for all wind velocities, since this simulations does not include feedback on the binary parameters.
The torque from the gas becomes more negative the lower the wind velocity is. As pointed

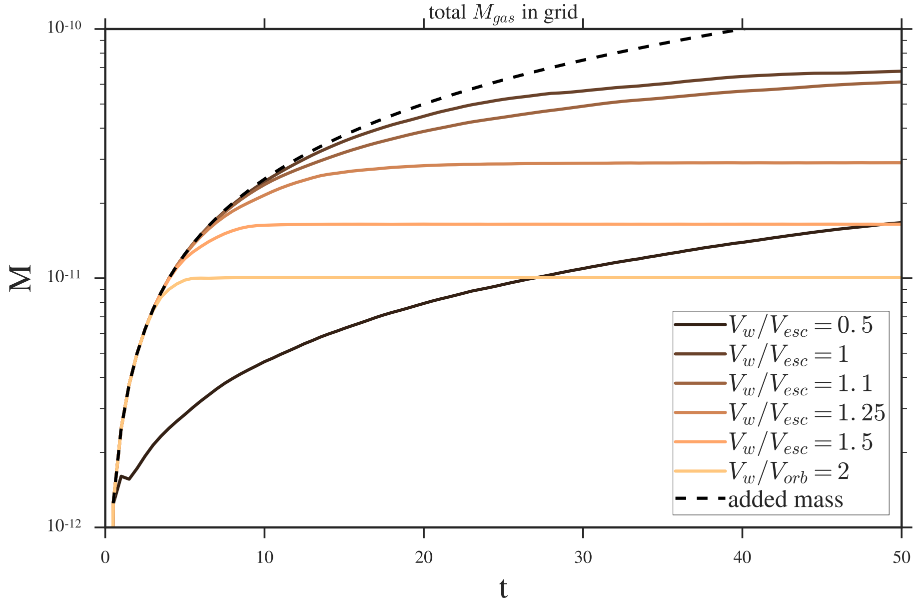

Figure 3: Total mass present in the grid as the wind moves outward. Added mass (dashed line) is the total gas mass injected at the inner boundary integrated over time. Solid lines are total gas mass present in the grid at time t for different wind velocities. For the fast winds (above 1.25) the mass flows outwards and reaches a steady state, where inflow and outflow are balanced. These winds are higher than escape speed at the injection site, but for the slower winds, mass falls back onto primary and is overwritten.

out earlier more material is present around the binary for slow winds, which will create denser wakes giving higher dragforces. For the slow winds the torque has periodic wiggles. Some fits the period, while others are subperiodic and could be coursed by disk formation around the companion.

By taking a time average of the torque after it reaches steady state around $t = 20$, we can get the total change in angular momentum $\dot{J_{net}}$ for the different wind velocities. This is plotted in figure 5.
The total angular momentum los will always be more negative than the analytical solution, Jeans mode. There will always be some interaction between the binary and the gas.

Finally with $\dot{J_{net}}$ we can calculate $\dot{a}$ for the orbit. Figure 6 shows this calculation.
For fast winds the orbit is still widening, since $\dot{a}$ is positive, but not as fast as expected from the jeans mode. For slow wind velocities the sign og $\dot{a}$ is flipped, and the orbit should instead shrink.

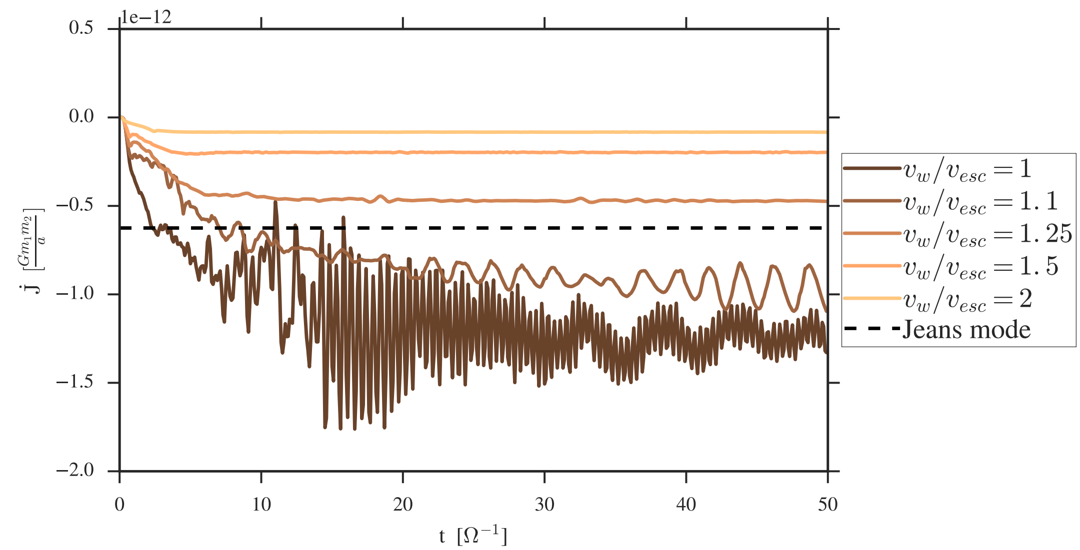

Figure 4: Total torque on the stars from gas and angular momentum lost directly from the donor, Jeans mode. The slower the wind, the more gas is present in wakes around the binary. The more gas in the wakes, the higher is the torque on the stars.

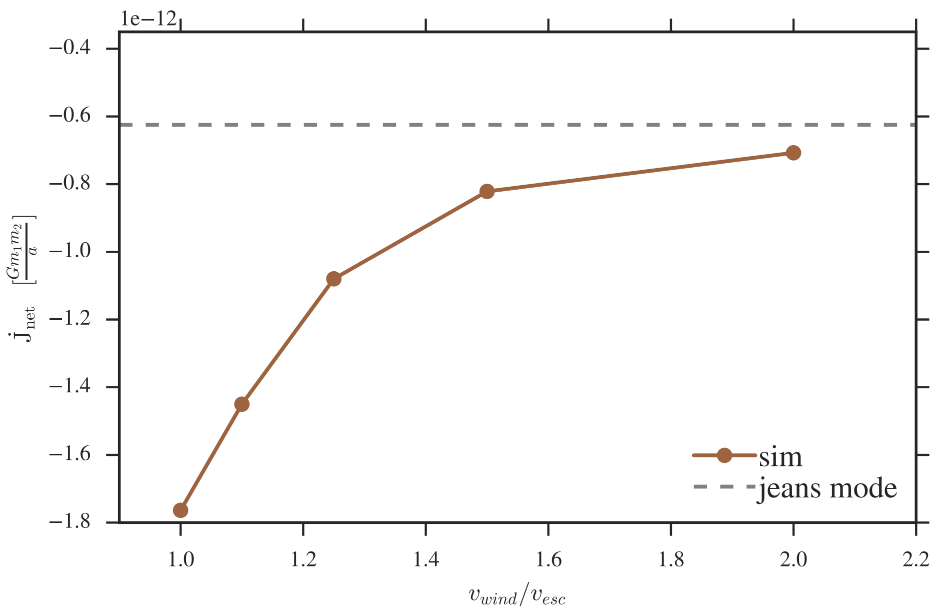

Figure 5: Total change in angular momentum. The value is always lower than the estimate from Jeans mode, because there always is some interaction with the gas.

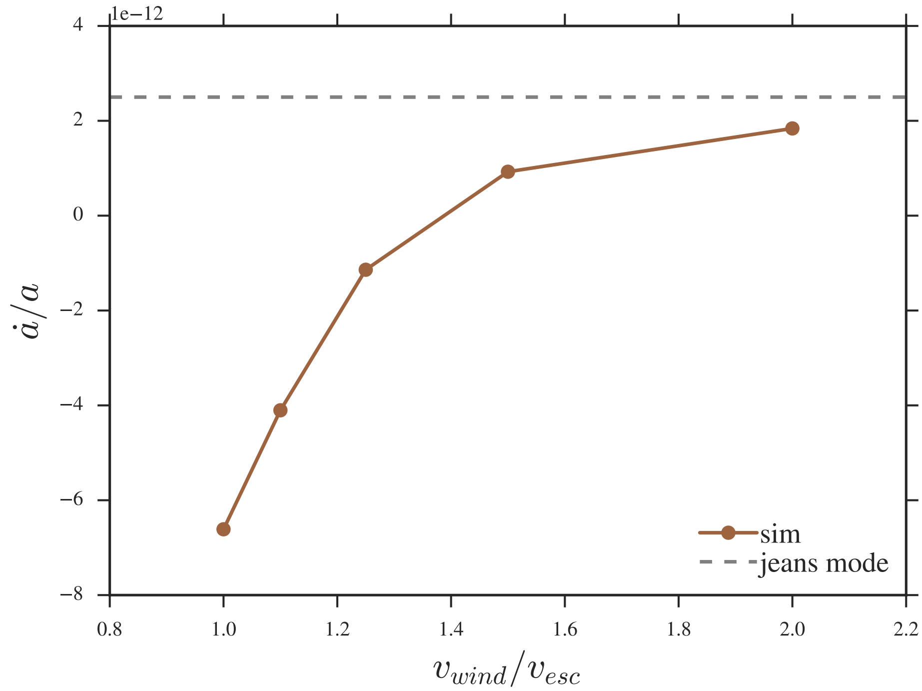

Figure 6: $\dot{a}$ for the different wind velocities. For fast winds the orbit is widening, for slow winds the orbit shrinks.

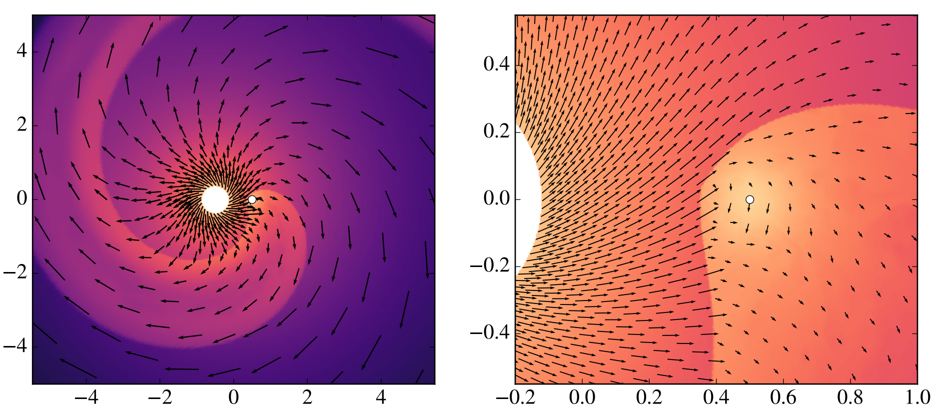

Figure 7: Two extra pretty plots: Velocity structure in the gas for the fast wind $v/vesc = 1.5$. Material is moving away from the binary, only blocked at the bow shock of the companion. The wind direction in the wake is more circular.

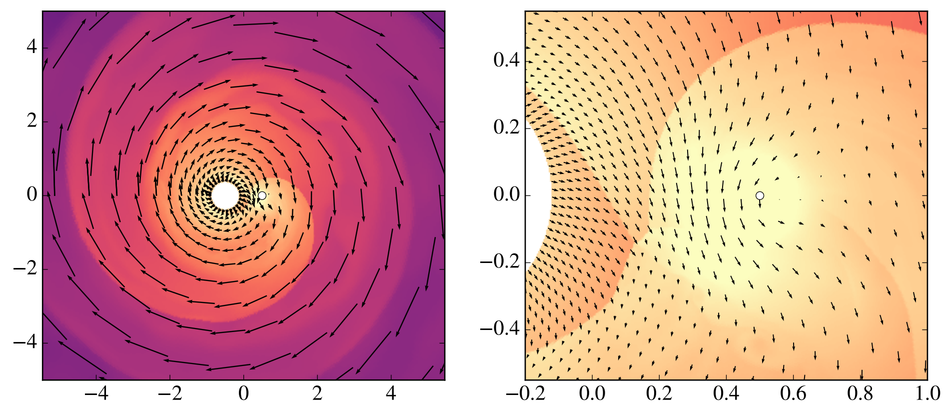

Figure 8: Two extra pretty plots: Velocity structure in the gas for the slow wind $v/vesc = 1$. Here all material is in wakes, so all material is moving circularly.