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The Kepler Catalog – A Tale of Evaporation

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ABSTRACT

The *Kepler* mission has discovered thousands of super-Earths and sub-Neptunes orbiting close to their parent stars. At least some of these low-mass planets are known to contain voluminous H/He atmospheres, which can evaporate away with time. Since a continuum of models with different core compositions, core masses, and envelope thicknesses can fit the mass and radius of a planet, the mass–radius distribution observed today is degenerate, and hence provides only weak constraints on composition. It has been argued that by accounting for evaporation one can constrain the evolution of the atmospheres and thereby break the degeneracies. In previous work, a statistical method was developed to retrieve the initial conditions of the Kepler-36 system. In this work, we apply this method to a different region of the planet mass-radius parameter space, by studying HIP 116454b, a $2.53 \pm 0.18\,\mathrm{R}_\oplus$, $11.82 \pm 1.33\,\mathrm{M}_\oplus$. For its current mass and radius, HIP 116454b is consistent with either an ice-rich solid planet with no envelope, or a less massive, denser core surrounded by a thick atmosphere. We statistically constrain the mass and composition of the core and the mass fraction and initial cooling time of the envelope of HIP 116454b by running planetary evolution models that include evaporation. We find that accounting for evaporation yields consistent results with previous studies. We find that the preferred initial properties for HIP 116454b are a rocky core with modest H/He atmosphere ($\sim 1\%$ by mass); approximately $\sim 50\%$ of the atmosphere is lost due to evaporation over the system lifetime. We also confirm the robustness of the method used by extending its domain of validity to larger and denser planets, and speculate on how the method will work in different regions of parameter space.

1. Introduction

The *Kepler* mission has altered our understanding of exoplanets by detecting thousands of planets close to their parent stars. The majority of these close-in planets ($\leq 0.3\,\mathrm{au}$) are small ($\leq 3\,\mathrm{R}_\oplus$). They have been discovered by the transit method, which provides a measurement of a planet's radius. Recent follow-up analysis using Transit Timing Variation (TTV) and radial velocity observations have also given us measurements of planetary masses. These results make it possible to study the mass–radius diagram (MR diagram) of small planets (e.g. Wu & Lithwick 2013; Weiss & Marcy 2014; Wolfgang et al. 2016). These studies of the planetary mass-radius distribution have shown that roughly half of them are inconsistent with a solid body and hence have a voluminous, volatile atmosphere on top of their solid core. Indeed, Wolfgang & Lopez (2015) argued that the dominant planetary structure we see today is a solid rocky core overlain with an H/He envelope that has a mass fraction of a few percent.

Since the majority of discovered exoplanets are close to their parent star, any H/He atmosphere they possess is expected to be undergoing evaporation, driven by stellar high energy photons (e.g. Lammer et al. 2003; Yelle 2004; Murray-Clay et al. 2009) and indeed planet evaporation has been observed to be happening for several massive Jovian planets(e.g. Vidal-Madjar et al. 2003). Owen & Jackson (2012) showed that while Jupiter-like planets experience evaporation, they are able to retain essentially all of their initial H/He envelope because of their deep gravitational wells.

However, Owen & Wu (2013); Lopez & Fortney (2013) showed that lower-mass planets ($\lesssim 20\ \mathrm{M}_\oplus$) are much more susceptible to evaporation. The fraction of their initial H/He atmospheres that they can retain decreases from almost 100 % for planets orbiting with periods greater than 20 days to 0 % for planets with periods less than ~ 3 days. This result depends sensitively on the mass of the solid core.

Super-Earth and sub-Neptune planets with masses between ~ 2 and $\sim 20\,\mathrm{M}_\oplus$ orbiting with periods between 3 days and 20 days are massive enough to have an initially large envelope, yet light enough to allow for significant evaporation. As a result, they can lose a significant fraction of their initial envelopes over time. In this work we refer to the aforementioned planets as "Kepler planets".

The MR diagram for the Kepler planets shows degeneracies, since at a given observed mass and radius, different planetary structure models can fit the data (e.g. Rogers & Seager 2010). The physical origin of this degeneracy comes from the fact that less-massive dense cores with larger atmospheres have identical masses and radii to those planets with than less dense, more massive cores with smaller atmospheres. The observed radius of the planet (which is the radius of the core plus the thickness of the atmosphere) can compensate the different radii of the cores by including a slightly different atmosphere.

This degeneracy is preventing us from using the observed data to full test planetary formation models, which include predictions of planetary composition. To break the degeneracy, one needs to add extra constraints on the planetary structure models, by either getting more information about the planet today (e.g., spectroscopic data – something almost impossible for the Kepler-planets as they are too far away) or by studying the evolutionary history of the planets in order to rule out certain planetary structures today. Owen & Wu (2013) showed that the Kepler planets may have undergone dramatic evaporation during their history. More recent work (Lopez & Fortney 2014; Owen & Morton 2016) has shown that accounting for evaporation in evolutionary models is able to place additional constraints on the planetary structure models today, as well as on their initial properties.

One of the aims of using planet evolution models is the possibility of linking together observations (at the current epoch) to formation theories by providing a distribution of initial models that fit the observed MR diagram today. The goal is to describe these initial models in terms of an initial envelope mass fraction–core mass diagram (the "Owen-diagram"). The initial planetary parameters can be compared to formation theories, such as Lee & Chiang (2015). The first statistical study of the initial conditions of a system has been carried out by Owen & Morton (2016). They retrieved the full posterior distribution of the Kepler-36 system containing two planets (b and c) in a 7:6 mean motion resonance. Kepler-36b is a $4.4\,\mathrm{M}_\oplus$ super-Earth and Kepler-36c a $7.3\,\mathrm{M}_\oplus$ *puffier* planet. This work presented the first framework to retrieve the posterior distribution of the initial properties for Kepler planets; they did this in terms of four parameters: the core mass, core composition, initial H/He envelope mass fraction, and the initial cooling time. This method has been applied and tested in the case of a small Kepler planet (Kepler-36b), and for a larger planet (Kepler-36c) with an envelope mass fraction today of about 10 %. Our current work extends the validity of the framework by extending its range of application to a more massive planet, HIP 116454b, which is expected to have a thin atmosphere today (Vanderburg et al. 2015). The locations of HIP 116454b and Kepler-b and c are shown in figure 2 today. Figure 2 shows the mass and radius relation as given by Fortney et al. (2007). Most of the planets have only had their radius measured by the Kepler mission, so that the mass is given by Wright et al. (2011). In this research note, we report the posterior distribution of the initial conditions of the planet HIP 116454b and show that such retrieval can also be applied to larger ($\sim 10 - 15\,\mathrm{R}_\oplus$) planets that undergo less evaporation, at the cost of weaker constraints on the initial conditions. We also update the core mass – initial envelope mass fraction diagram.

2. Overview

HIP 116454 is a bright K1-dwarf with mass $M_\star = 0.775 \pm 0.027\,\mathrm{M}_\odot$ (Vanderburg et al. 2015). A transiting planet, HIP 116454b has been detected with a measured radius of $R_p = 2.53 \pm 0.18\,\mathrm{M}_\oplus$. Using the radial velocity technique the mass of the planet has also been measured, and found a value of $M_p = 11.82 \pm 1.33\,\mathrm{M}_\oplus$, with an orbit of 9.1 days. According to Vanderburg et al. (2015), HIP 116454b's composition is consistent with either a low-density solid body made of $\sim$ 75%H_2O and 25%$MgSiO_3$ with no H/He envelope or with a smaller core ($\sim 1.8\,\mathrm{R}\oplus$) and assuming a H/He envelope and a negligible albedo, the Lopez & Fortney (2014) models predict that the envelope

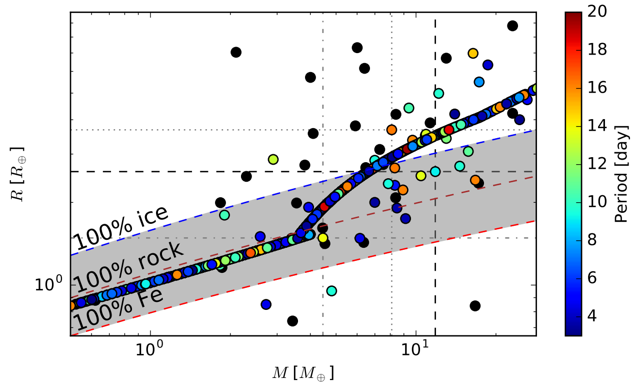

Fig. 1. MR diagram for the Kepler planets. The red, brown and blue dashed lines indicate the positions of solid planets made respectively of 100% Fe, 100% rock and 100% ice using the models of Fortney et al. (2007). The data are from the Kepler catalog. The color encodes the period of planets between 3 and 20 days. The location of HIP 116454b is shown by the dashed lines. The locations of Kepler-36b and c are indicated by the dash-dot and dotted lines respectively.

accounts for 0.5 % of the planetary mass, and the thickness of the atmosphere is $0.35\,\mathrm{R}_\oplus$.

If it were purely solid, HIP 116454b would be one of the "H_2O rich" planets described in Zeng & Sasselov (2014). Various evidence points to HIP 116454 having an age of approximately 2 Gyr. Using relations from Mamajek & Hillenbrand (2008), the R'_{HK} level indicates an age of 2.7 Gyr and the rotation indicates an age of 1.1 Gyr, if the stellar rotation period is indeed close to 16 days. If HIP 116454's age is indeed about 2 Gyr and the planet lacks a gaseous envelope, then it is likely to have water in plasma phases near its water-silicate boundary (the bottom of the H_2O layer), while if it is slightly older ($\sim$ 3 Gyr or more), or has a faster cooling rate, it could have super-ionic phases of water present in its interior.

3. Method

We assume the initial models contain a solid core, where we take the mass and radius from the models of Fortney et al. (2007). In this treatment, the composition of the core is characterized by a single parameter C that takes values in the range $-1 \leq C \leq 1$. A pure Fe core has $C = -1$, a pure $MgSiO_3$ has $C = 0$ and an ice core has $C = 1$. Following Owen & Morton (2016), the model assumes that the solid core is surrounded by an H/He envelope. All other chemical species are neglected since they are not expected to play an important role in the evolution or structure of the bulk atmosphere. The atmosphere is initially heated with a fixed internal heat flux which balances the PdV work of the envelope's contraction. The top of the atmosphere is also irradiated by the star. The incoming flux from the star is initially set to be the stellar flux at an age of $t_\star = 3\,\mathrm{Myr}$, consistent with the time at which the planet is released from the disk it forms within. The initial models are assumed to be in a steady state, so we exclude any model with a total radius close to the Bondi radius; here we take the critical value to be $R_p \approx 0.2 R_\mathrm{b} = 0.2 \times 2GM_p/c_s^2$, where c_s is the sound speed, G is the gravitational constant, M_p and R_p

are the mass and radius of the planet (c.f. Owen & Morton 2016). Models with $R_p \gtrsim 0.2R_\mathrm{b}$ could not reach a steady state once the core mass, core composition, envelope mass fraction, and internal heat flux are set because the envelope gas can escape, even in the absence of high energy irradiation, hence decreasing the envelope mass fraction (see Owen & Wu 2016, for a description of this process).

We place extra constraints on the formation history of the planet by constraining the initial cooling time, the Kelvin-Helmholtz time-scale, which is the time required for the H/He envelope to radiate away all its gravitational potential energy. This time scale is given by $t_\mathrm{kh} \sim (GM_pM_e/R_p)/L_p$, where L_p is the luminosity of the planet and M_e is the envelope mass.[1] We reject any initial model with a cooling time smaller than $\sim 1\,\mathrm{Myr}$. Models with a smaller cooling time would have contracted significantly before the disk dispersed. We also reject any model that has a cooling time longer than a few billion years. Models with such long cooling times correspond to planets with thin and dense atmospheres. Accretion is unlikely to leave a thin, dense and cold atmosphere because the contraction of the envelope is a major source of heating. It is also difficult to imagine how a cooling planet could have an initial cooling time greater than the age at which we see the system today (Owen & Morton 2016).

In addition to the planet evolving in time, the star also evolves, which in turn affects the planetary evolution. The radius and effective temperature of the star are computed using a standard model, which matches the observed properties of the star today, and is obtained using the Modules for Experiments in Stellar Astrophysics (MESA) computational package (Paxton et al. 2011, 2013, 2015). For the planetary evolution, the numerical code that we use to compute the evolution of the planets is a modified version of MESA, (Owen & Morton 2016). This model accounts for both X-ray and EUV driven winds, as prescribed in Owen & Jackson (2012); Owen & Wu (2013), as well as the evolution of the stellar bolometric irradiation of the planetary atmosphere. For an individual model, only the core mass and composition remain fixed during its evolution and the evolution of the planetary atmosphere is computed in a self-consistent manner. Following Paxton et al. (2013) and Owen & Morton (2016), we adopt a $F_\star - \Sigma$ model to account for the bolometric irradiation from the star. In this treatment, the stellar flux ($F_\star$) is absorbed by a layer (with thickness Σ) on top of the atmosphere, where the value of Σ corresponds to the surface density above the optical photosphere.

We then build a comprehensive grid of models filling the initial parameter space and evolve the models during 2 Gyr (which is an estimate of the age of HIP 116454). The uncertainties in the age of the system are large, but since the mass loss is dominated by the first few hundred million years, the final results are not overly sensitive to the uncertainties in the age of the system.

Using these forward models and the posterior distributions of the observed planet's mass and radius, we are able to infer the posterior distributions of the initial planetary composition parameters: the core mass, core composition, initial envelope mass fraction, and initial cooling time.. This approach to inference, in which a model is conditioned on quantities of which the observations are themselves uncertain, is known as multi-level or hierarchical inference (see, e.g., (Hogg et al. 2010; Demory 2014; Foreman-Mackey et al. 2014; Morton & Winn 2014; Wolfgang & Lopez 2015; Wolfgang et al. 2016)).

[1] Note that in our numerical calculations we calculate t_kh exactly as $|U|/L_p$, where U is the total gravitational binding energy of the envelope.

4. Results

Our approach has allowed us to successfully model the evolution of HIP 116454b and thus constrain its initial properties. The posterior distributions of HIP 116454b are shown in Figure 2 for the core mass (left panel), core composition (middle left panel), initial envelope mass fraction (middle right panel) and initial cooling time (right panel). The numbers reported on the figure are the 16%, 50% (median) and 86% quantiles. If the distribution was a normal distribution, these values would match with the mean and $1\,\sigma$ error bars. The maximum of likelihood for the core mass ($M_c = 11.99^{+1.13}_{-1.06}\ \mathrm{M}_\oplus$) is consistent with the observed mass today ($M = 11.82 \pm 1.33\ \mathrm{M}_\oplus$), assuming a small envelope mass fraction. The constraint on the core composition shows that our calculations of the planet's evolution favour denser cores, i.e., (rocky) to iron-rich ones ($C = 0.04^{+0.30}_{-0.39}$). We also constrain the initial envelope mass fraction to be smaller than $\sim 2\%$ ($\log X = -2.25^{+0.58}_{-0.85}$), and also prefer models that had *some* initial H/He envelope. We get weak constraints on the initial cooling time, even though the formation history is slightly more favourable to cooling times shorter than 1 Gyr.

The position of HIP 116454b in the "Owen diagram" is presented in Figure 3 along with the results from Owen & Morton (2016) for Kepler-36b and c. The blue solid line indicates the envelope mass fraction as a function of core mass, after billions of years of evolution, using the fitted mass-radius relation measured *today* from Wolfgang et al. (2016) interpolated using the Lopez & Fortney (2014) structure models to get the envelope mass fraction. The dashed and dotted lines show the physical $1-\sigma$ and $2-\sigma$ spread on the MR diagram. The initial conditions of HIP 116454b are consistent within $2-\sigma$ with the observed mass-radius relation today. The final envelope mass fraction is $\log X_\mathrm{today} \lesssim -2.3$ (0.5%) with an upper limit of ~ -1.69 (2%). Thus, HIP 116454b has lost roughly 60% of its initial H/He envelope.

Our results have placed statistical constraints on the initial composition of HIP 116454b, a planet with a mass similar to Neptune, but with a thinner atmosphere. Our results indicate that HIP 116454b was initially composed of a large core with a thin envelope comprising $\sim 1\%$ of the mass, with an upper limit of $\sim 3\%$. The evolution calculations rule out any planet model with a final envelope more massive than 2%, but do not rule out models where the H/He envelope has been entirely evaporated. We thus argue that HIP 116454b is a super-Earth that has lost most of its primordial atmosphere (or perhaps all of it).

5. Discussion

The method developed by Owen & Morton (2016) has already been tested successfully for two planets smaller than HIP 116454b. One of these planets is rocky today, whereas the other has a large H/He envelope. We have shown here

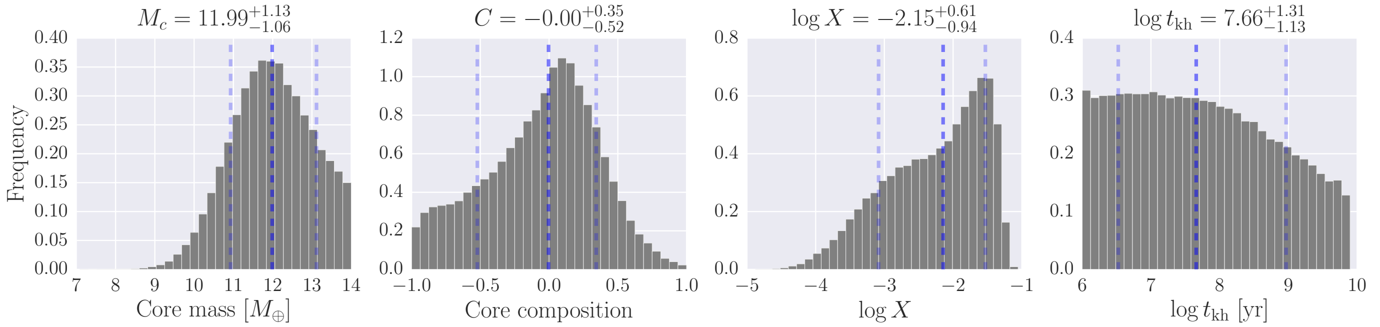

Fig. 2. Distributions of the expected initial parameters of HIP 116454b. The dashed vertical lines are the position of the 16%, 50% (median) and 86% quantiles. The first (from left to right) panel shows the core mass, the second shows the core composition, parametrised from –1 (pure iron), 0 (pure rock) to 1 (pure ice) where –0.5 would be a 50% iron/rock mixture and 0.5 would be a 50% ice/rock mixture. The third panel shows the initial envelope mass fraction (X) and the final panel shows the initial cooling time (Kelvin-Helmholtz time-scale).

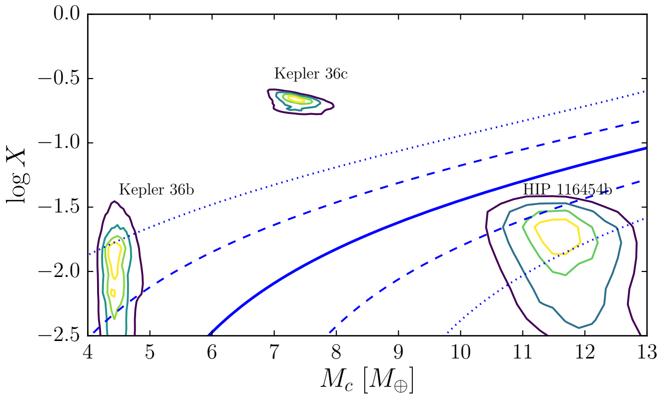

Fig. 3. Core mass – envelope mass fraction diagram with the results from Owen & Morton (2016) for Kepler-36 and our new results for HIP 116454b. The contour levels are0.5, 1.0, 1.5, 2.0σ, from yellow to purple.

that evolutionary models that include evaporation are still able to constrain the initial conditions and the final planet properties for the case of a larger, denser planet. This success comes at the cost of poorer constraints on the envelope mass-fraction and the cooling time of the planet, as illustrated in Figure 3. We speculate that this method has an upper limit of about 20 $\mathrm{M}_\oplus$. Above this mass threshold, our technique cannot infer extra information from evaporation and cannot break the MR diagram degeneracies. Moreover, we confirm that our method is able to extract information from planets having a thin atmosphere, with an envelope mass-fraction of at most a few percent today.

We hypothesise that on the "puffy" side of the MR diagram, this method will be able to yield precise predictions of the initial envelope mass fraction and cooling time. Puffy planets are more sensitive to the magnitude of the internal heat flux: as the heat flux grows, the atmosphere expands and grows to radii closer to the Bondi radius, thereby making evaporation easier. We can use these insights to hypothesize how the kind of constraints evaporation models will yield for such planets. Low-density planets with fast cooling times and large internal heat flux are likely to evaporate significantly. If the planet today has a large envelope, we should be able to get an upper limit on the initial cooling time. This limit comes from the largest internal heat flux (smallest cooling time) consistent with a initial planet radius that is still a "bound" planet and a final envelope mass fraction consistent with observed data.

In future work, this hypothesis will be checked using the Kepler-18 system, a system composed of three puffy planets of 6.9 $\mathrm{M}_\oplus$, 17.2 $\mathrm{M}_\oplus$ and 16.5 $\mathrm{M}_\oplus$ orbiting their star with orbital periods of 3.5, 7.6 and 14.9 days respectively.

The retrieval of the initial conditions of Kepler-36b and HIP 116454b shows that our method is able to get reasonable constraints on the core composition, as well as limits on the initial envelope mass fraction for dense planets. Planets with thin envelopes have a gravitational well dominated by the core's bulk composition, as the top of the envelope is deep in the potential well. The efficacy of evaporation is then driven by the depth of the potential well that the upper atmosphere experiences. This issue can be studied by comparing the contraction time-scale (the Kelvin-Helmholtz time-scale) to the evaporation time-scale ($\lesssim$ 1 Gyr). If the Kelvin-Helmholtz time-scale is smaller than the evaporation time-scale, the planet has time to reach an equilibrium temperature that is independent of the initial internal heat flux before evaporation is not important. Because the planet has time to "forget" its initial heat flux, the strength of evaporation is largely independent of this cooling time. This feature explains the mostly flat distribution of cooling times observed for Kepler-36b and HIP 116454b. If the cooling time-scale is longer than the evaporation time-scale, then the initial internal heat flux sets the equilibrium temperature of the planet and this temperature can be considered as constant during the evaporation. Higher temperatures result in less dense envelopes that are easier to evaporate, and thus effectively boost the efficiency of evaporation. This explains the observed cut-off of the initial Kelvin-Helmholtz time at about 1 Gyr for Kepler-36b and HIP 116454b.

6. Conclusions

In this work we have used planetary evolution models that include evaporation to follow the evolution of the planet HIP 116454b. By considering those models which are consistent with the observed mass and radius of HIP 116454b today, we have statistically constrained the initial properties for HIP 116454b. Our results provide constraints on the core composition as well as limits on the initial envelope mass fraction of HIP 116454b. The main results of our study are summarised below:

1. We validate the method developed in Owen & Morton (2016) by using evolutionary models which include evaporation to estimate the initial conditions for exoplanets in the mass range $\sim 10-20\mathrm{M}_\oplus$. This range expands the parameter space explored previously.
2. We found constraints on the core mass, core composition, and initial envelope mass fraction for HIP 116454b. We find that HIP 116454b is more likely to have a rocky/earth-like core than a strongly ice-rich core. As such, it is likely to have been born with a $\sim 1-2\%$ H/He envelope mass fraction of which $\sim 50\%$ remains. However, the models provide essentially no constraints on the initial cooling time for HIP 116454b.
3. While we have updated the core mass – initial envelope mass fraction diagram, no clear trend can be identified with only three points. This hints at the fact – unsurprisingly – that there is likely a large scatter in the underlying distribution.

For HIP 116454b we find a formation scenario where the planet accretes an initial envelope mass fraction on the order of a few percent, or less, roughly consistent with previous results (Vanderburg et al. 2015). Since the initial H/He envelope has little mass, the initial cooling time is poorly constrained: Low-mass atmospheres can support a large internal heat flux before reaching a size that is a significant fraction of the planet's Bondi radius (at which point the envelope would become unbound), allowing a wide range of cooling times.

The results for HIP 116454b are consistent with the calculations performed by Wolfgang et al. (2016), but more systems need to be studied. Because evaporation is most efficient for planets that have thick atmospheres, we expect to get better constraints on the initial envelope mass fraction as well as on the initial cooling time for initially *puffier* planets. On the other hand, we can obtain good constraints for the core composition for planets with thin atmospheres like HIP 116454b or Kepler-36b.

To be able to confirm or reject this hypothesis, we will explore the region of the core mass versus envelope mass fraction for envelope mass fractions larger than 30%. In follow up work we hope to do this for the Kepler-18 system. Combining the study presented here for HIP 116454b and our future study of Kepler-18, we will be able to test this hypothesis and this will be presented in a future peer-reviewed publication in next year.

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